SU(N) gauge theories with symmetric-rep fermions

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with

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Part I. $SU(N)$, $N = 2, 3, 4$, with $N_f = 2$ two-index sym rep Dirac fermions:
"continuum" issues; physics results

Part II: Fat (nHYP) links and what they do for us
$\Rightarrow$ It's a good idea to try DWF with fat links

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Walking [extended] Technicolor

- Confinement & Chiral symmetry breaking are lost for $N_f > N_{\text{critical}}$
- "Walking" theories are found just below the conformal window
- Condensate enhancement needed $\Leftrightarrow$ mass anomalous dimension $\gamma \approx 1$
- Natural candidates: $N_f = 2$ higher-irrep fermions [Sannino *et al.*]

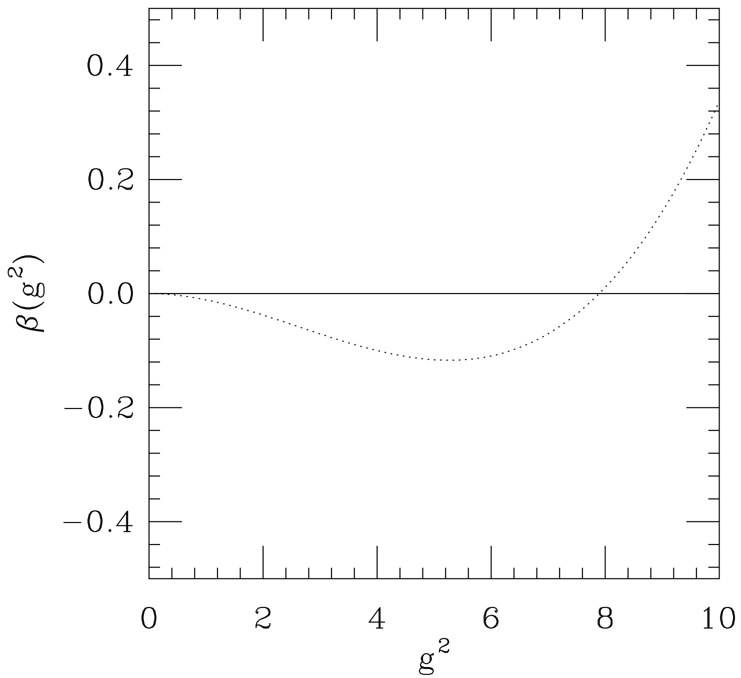
Schrödinger functional

- Induce background field thru boundary conditions in L^4 box
- Measure $1/g^2(L)$ from response to small change in boundary conditions
- Extract mass anomalous dimension γ from scaling of pseudoscalar renormalization constant Z_P, on the same lattices

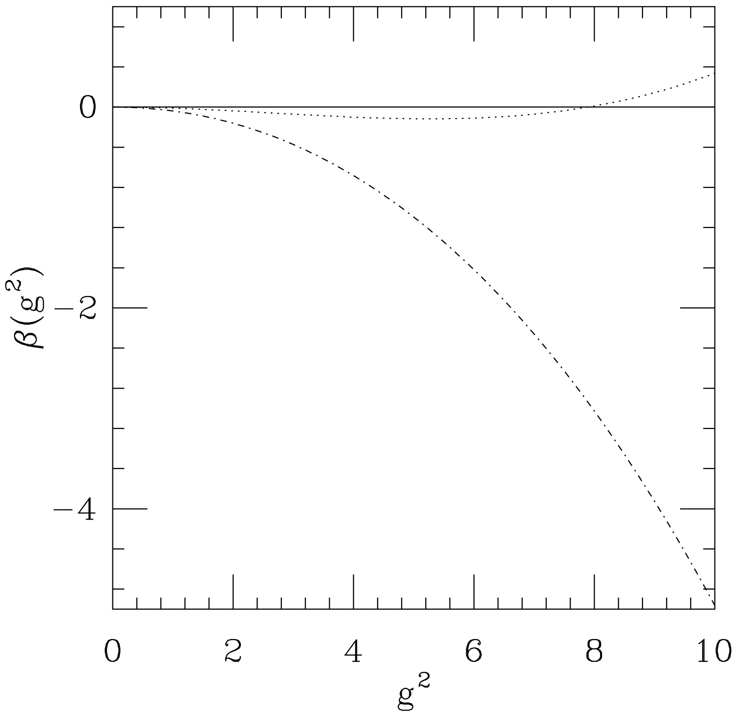
Life inside (or near) the conformal window

Two-loop beta function, SU(2) with $N_f = 2$ of:

adjoint



fundamental

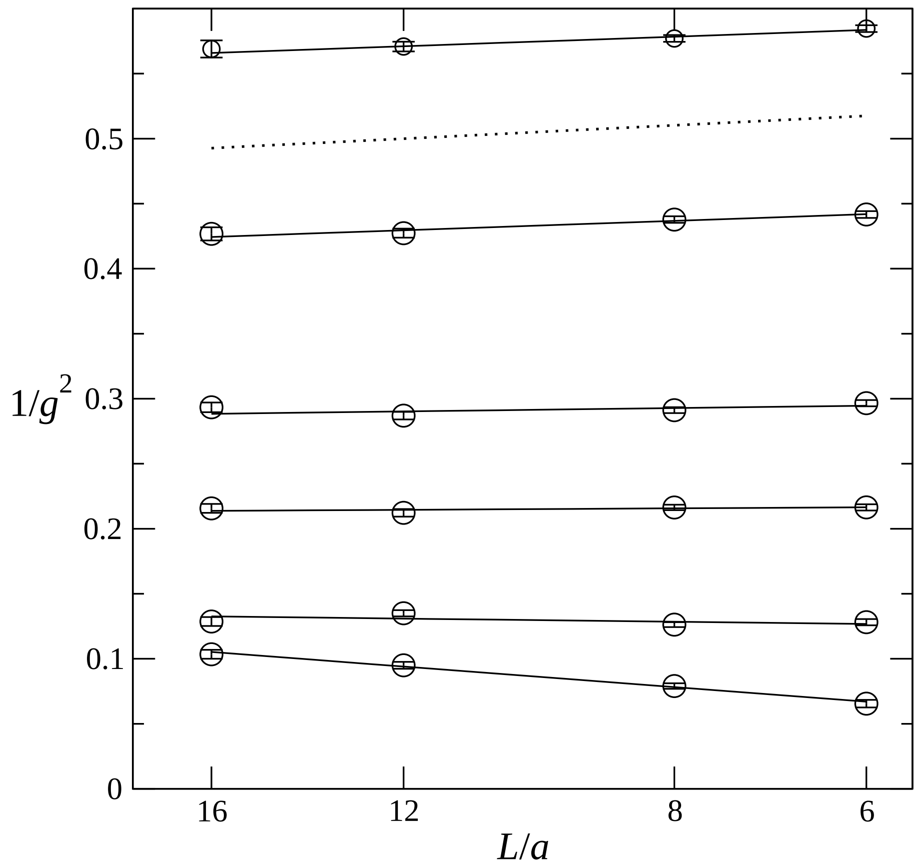


$\Rightarrow$ Strategy of ALPHA developed for QCD; performs poorly here

Nearly constant beta function

define: $\tilde{\beta}(1/g^2) \equiv \dfrac{d(1/g^2)}{d\log(L)} = 2\beta(g^2)/g^4$

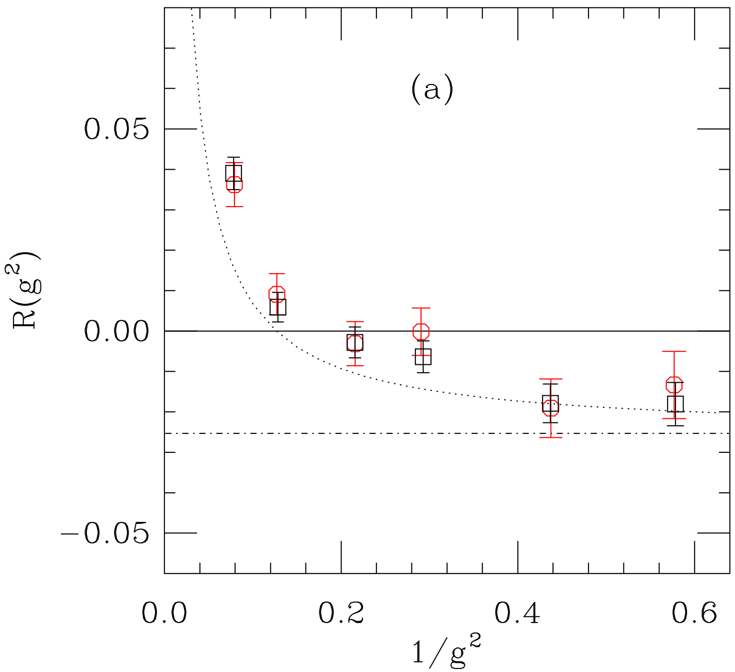
- Solution: $1/g^2 \simeq a + b\log(L)$
- Exact at one loop
- Treat dataset at each bare β as separate fitting problem
- slope = beta fn.
- Systematics: add $\log^2$ term, or remove smallest volume
- Slope changes sign $\Longrightarrow$ IRFP!



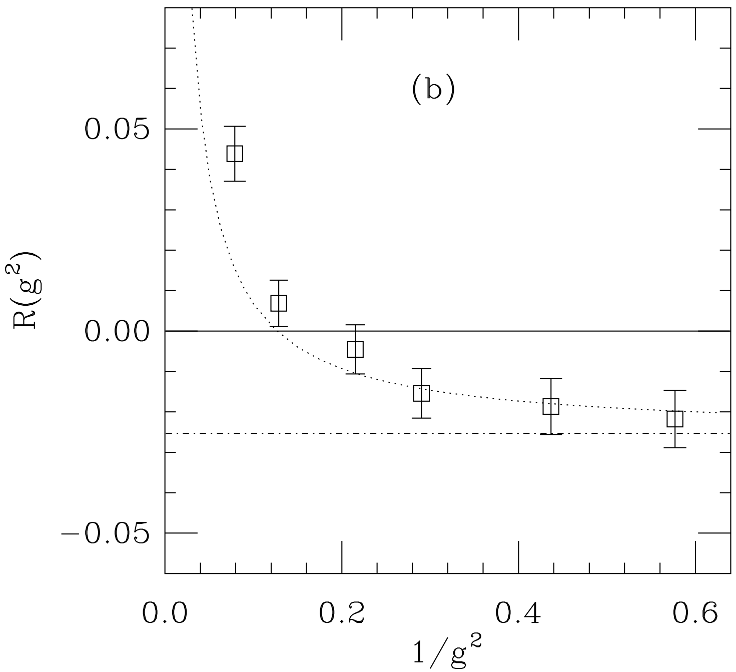
SU(2)/adjoint

Fitting $1/g^2$: $x = L/8$, Black: $L =$ 6,8,12,16, Red: $L =$ 8,12,16

$a + b\log(x)$



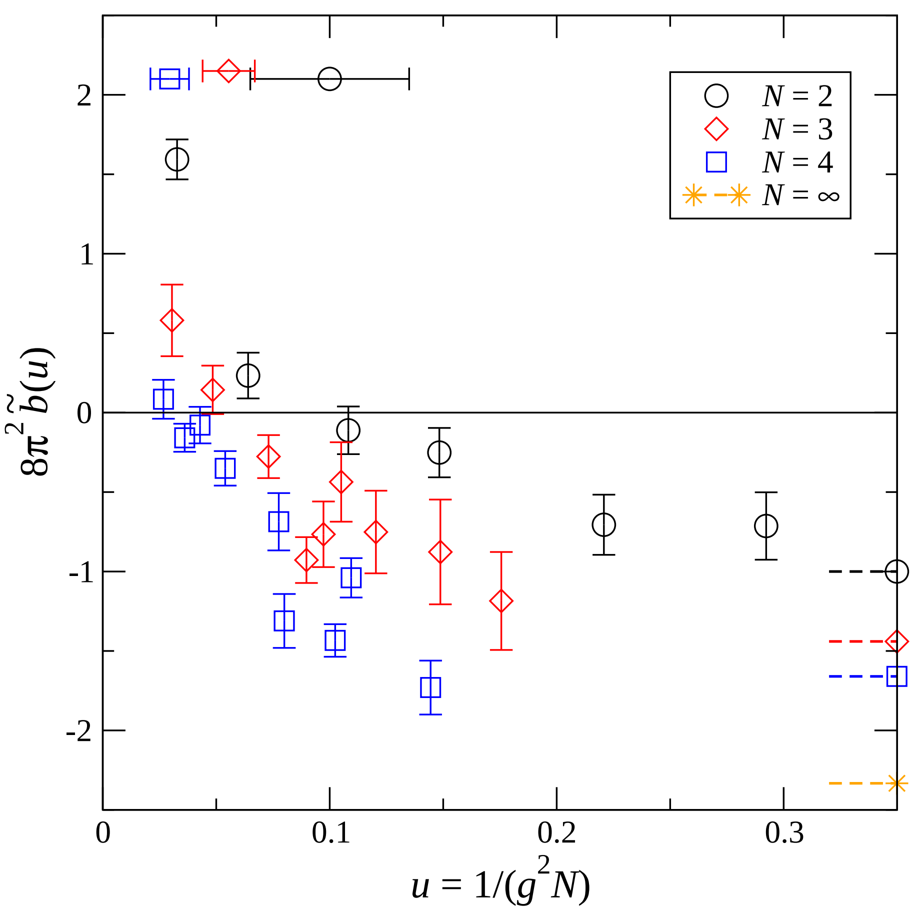
$a + b\log(x) + c\log^2(x)$



$$\boxed{1/g_*^2 = 0.20(4)(3)}$$

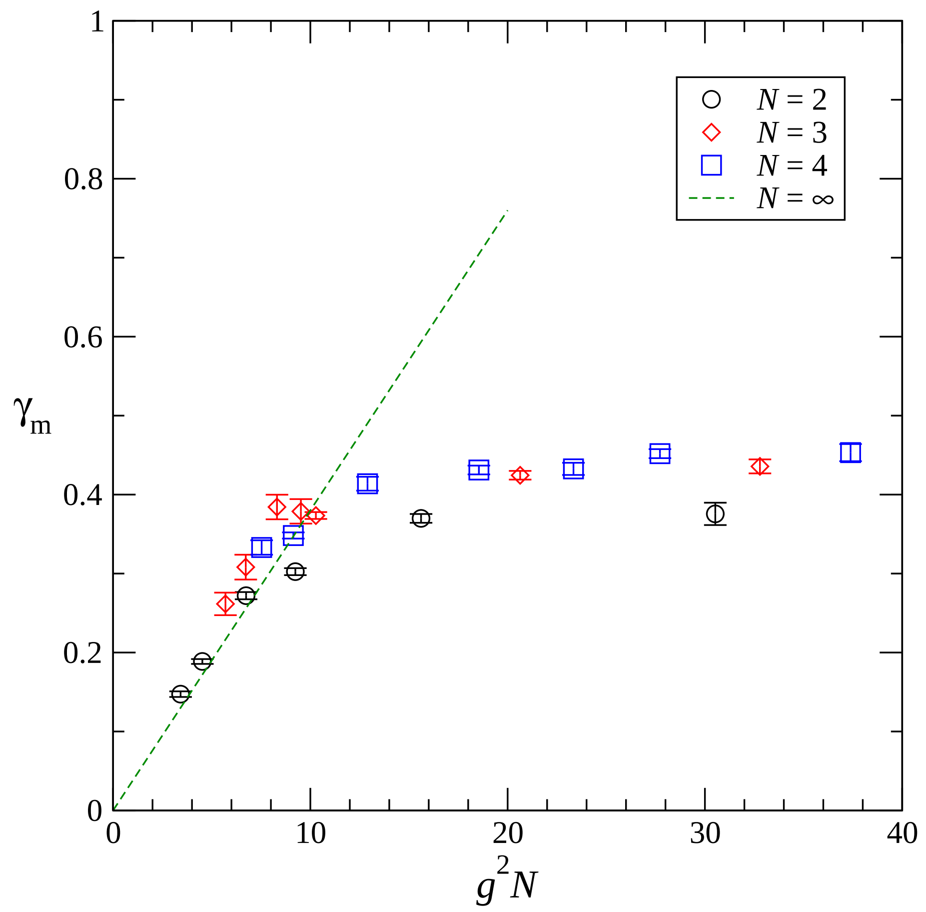
SU(2)/adjoint, SU(3)/sextet, SU(4)/decuplet

- Qualitatively agrees with $\beta^{\text{[2-loop]}}$
- Actual $\beta > \beta^{\text{[2-loop]}}$
- With increasing N_c, IRFP moves to stronger coupling, or disappears
- Consistent with no IRFP of $\beta^{\text{[2-loop]}}$ for $N_c \to \infty$ limit
- Can one use SU(4)/decuplet for walking technicolor?



Mass anomalous dimension

- Dashed line:
  one-loop for $N_c \to \infty$
- Remarkable universality
- Saturation: $\gamma \lesssim 0.45$
- Missed by analytic calculations!
- No good for walking technicolor



Part II. What nHYP links do for us

- Results from SU(3)/sextet

	β	$g^2_{SF}(L=6)$	κ_c	m_c	M
thin	6.0	~ 2	0.1610	-0.89	1.9
fat	5.8	1.9	0.1283	-0.10	1.1

Critical hopping parameter $\kappa_c = 1/(8+2m_c)$

Optimal Domain-Wall height $M = 1 + |m_c|$ (to be explained later on)

- Similar improvement for optimal clover coefficient (big enough that we've decided to set $c_{SW} = 1$)
- Fat links allow us to probe much larger $g^2(L)$

DWF primer

- 5-d Wilson fermions, supercritical (negative bare mass), with chiral 4-d fields located near the boundaries $s = 0$ and $s = L$

- Effective 4-dim operator: $D_{\text{eff}}(L) = 1 + \gamma_5 \tanh\big((L/2)a_5 H\big)$

- For $L \to \infty$ it becomes a GW operator: $D_{\text{eff}}(\infty) = 1 + \gamma_5 H/\sqrt{H^2}$

- DWF transfer matrix: $T = \exp(-a_5 H) = 1 - a_5 H_W + \cdots$ where $H_W = \gamma_5 D_W$ is the 4-d hermitian Wilson operator

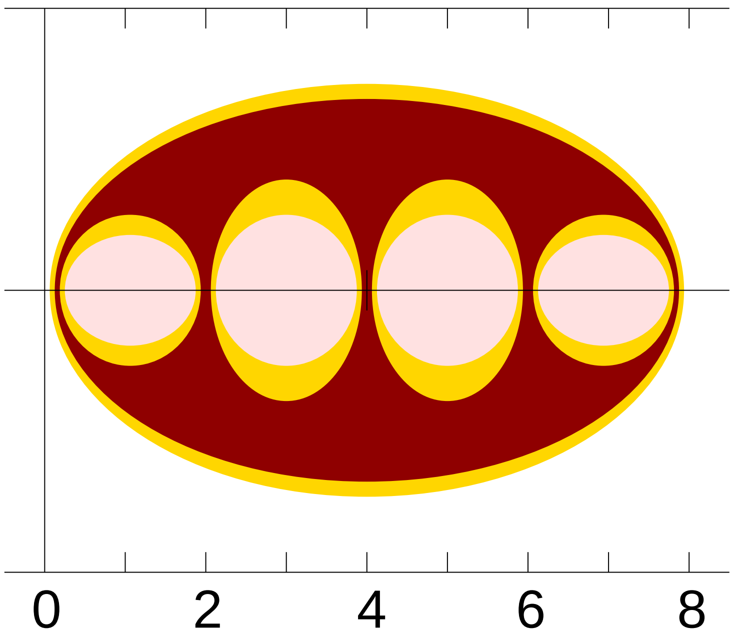
- Penetration into the 5-d bulk from near-unity eigenvalues of T, hence from near-zero eigenvalues of H_W

- PCAC: $\partial_\mu A_\mu = 2m_q J_5 + 2m_{\text{res}}\big(J_5 + \text{[more] lattice artifacts}\big)$

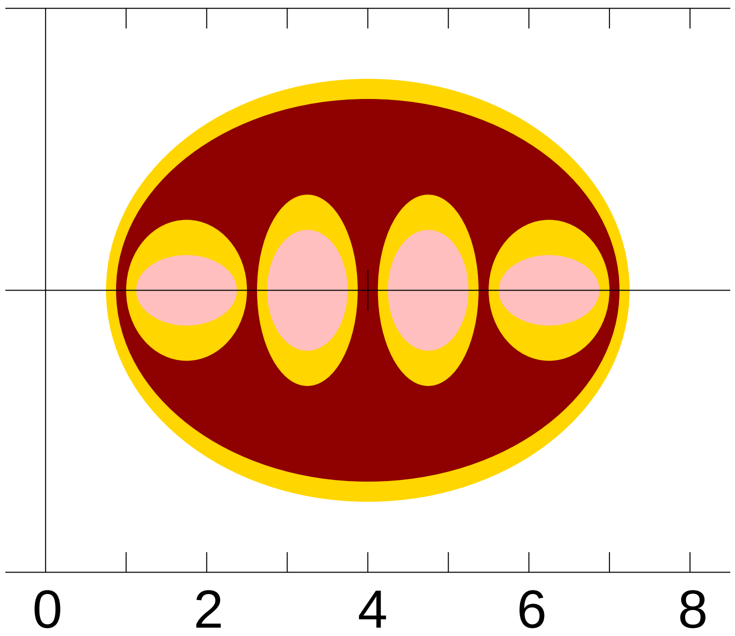
- Need $m_{\text{res}} \sim 10^{-3}$ (or smaller). Want L to be small.

spectrum of D_W in complex plane

- Optimal DW height $=$ center of leftmost "void"
- Fat links tame the Wilson spectrum: $m_c(\text{fat}) \ll m_c(\text{thin})$
- Expect many more ev's inside the thin-links "void": $m_{\text{res}}(\text{fat}) \ll m_{\text{res}}(\text{thin})$



fat links



thin links

Fat links and DWF

- DWF on staggered MILC lattices: [LHP Collaboration, . . .]
  thin: m_{res} "too big to measure"
  fat: $m_{\text{res}} \sim 10^{-3}$

- Locality
  – It's true that D_W is less local with fat links
  – nHYP links only mildly nonlocal: [Hasenfratz, Hoffmann & Schaefer]
  Smeared link depends only on thin links that share a hypercube with it

- GW operators are never ultra-local

- Relevant notion: exponential localization of $D_{\text{eff}}(L)$.
  – Depending on suppression of near-zero eigenmodes,
  in principle $D_{\text{eff}}(L)$ might be <u>more</u> local with fat links!

Summary

- $SU(N), \quad N = 2, 3, 4,$ with $N_f = 2$ two-index sym rep Dirac fermions are no good for walking technicolor because mass anomalous dimension saturates at $\gamma \lesssim 0.45$

- While several nice tricks are already being used to bring down m_{res} [improved gauge action, Dislocations Suppressing Determinant Ratio], and to speed up the inversion [Möbius], it's a good idea to try DWF with fat links

Walking (extended) Technicolor

For acceptable flavor physics, candidate theories should have:

- Chiral symmetry breaking
- Small S-parameter
- Avoid massless GBs (except those eaten by $W^{\pm}$ and Z)
- Condensate enhancement

$\Rightarrow$ need mass anomalous dimension $\gamma \approx 1$

$\Rightarrow$ this is expected for nearly conformal ("walking") theories

$\Rightarrow$ Natural candidates: $N_f = 2$ higher-irrep fermions [Sannino *et al.*]

Schrödinger functional

- Prescribe gauge field $A_k(L, \eta)$ on time boundaries: $t = 0$ and $t = L$

$\Rightarrow$ induce background color-electric field in the bulk

$$
\begin{aligned}
\Gamma(L, \eta) \equiv -\log(Z) &= \text{tree-level} + \text{one-loop} + \cdots \\
&= \left(\frac{1}{g_0^2(a)} + \frac{b_1}{32\pi^2} \log(L/a) + \cdots \right) S_{cl}(\eta) \\
&= \frac{1}{g^2(L)} \, S_{cl}(\eta)
\end{aligned}
$$

- Obtain $1/g^2(L)$ from variation w.r.t. η, which is an observable

- Extract mass anomalous dimension γ from scaling of pseudoscalar renormalization constant Z_P, on the same lattices

Soft gauge action

- Soft action: $S_{\rm gauge} = \beta \sum U_{\rm plaq}(\text{thin links}) + \tilde{\beta} \sum U_{\rm plaq}(\text{fat links})$

- Pushes 1st-order bulk transition back into stronger bare coupling

- SU(3)/sextet:
  – thin links: transition at $g^2_{SF} \approx 2.5$, lattice artifacts for $g^2_{SF} \gtrsim 2.0$
  – fat links, $\tilde{\beta} = 0$: transition at $g^2_{SF} \approx 5.0$, lattice artifacts for $g^2_{SF} \gtrsim 3.5$
  – fat links, $\tilde{\beta} = 0.5$: up to $g^2_{SF} \approx 11$ before running out of steam

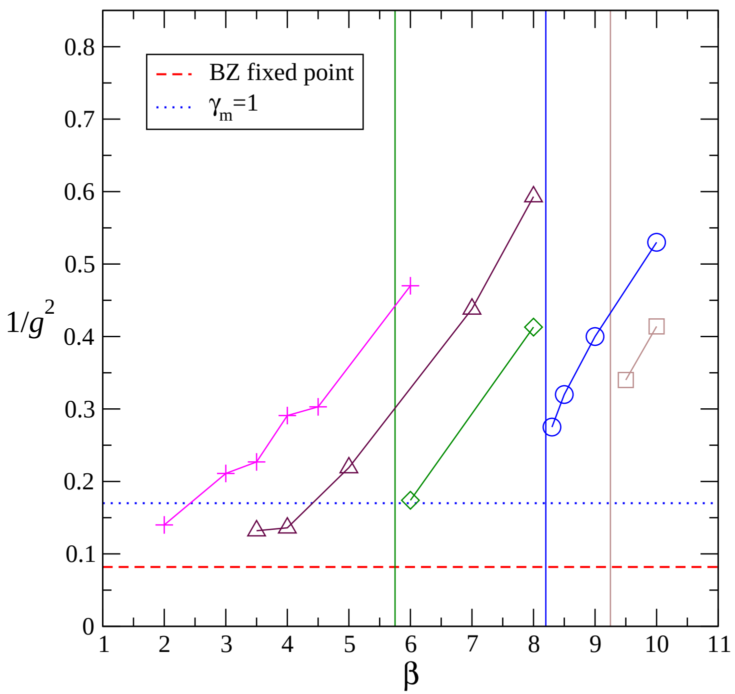
- Stabilize nHYP reunitarization step:

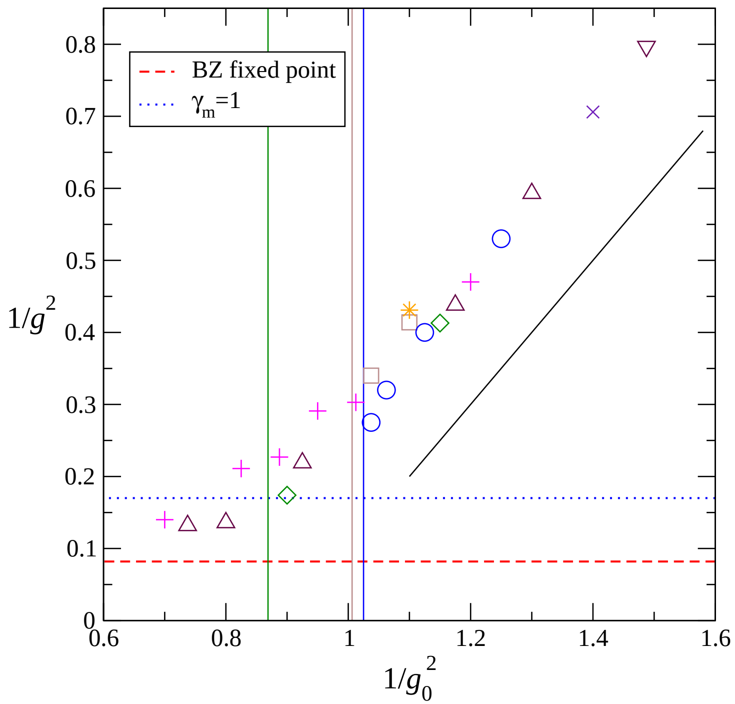
$$V_\mu = \Omega_\mu \left(\Omega_\mu^\dagger \Omega_\mu\right)^{-1/2} \qquad \Rightarrow \qquad V_\mu = \Omega_\mu \left(\Omega_\mu^\dagger \Omega_\mu + \epsilon\right)^{-1/2}$$

We use $\epsilon = 10^{-6}$

Soft gauge action: weak-coupling universality

bare coupling: $$\frac{1}{g_0^2} = \frac{\beta}{N}T(f) + \frac{\tilde{\beta}}{d_R}T(R),$$

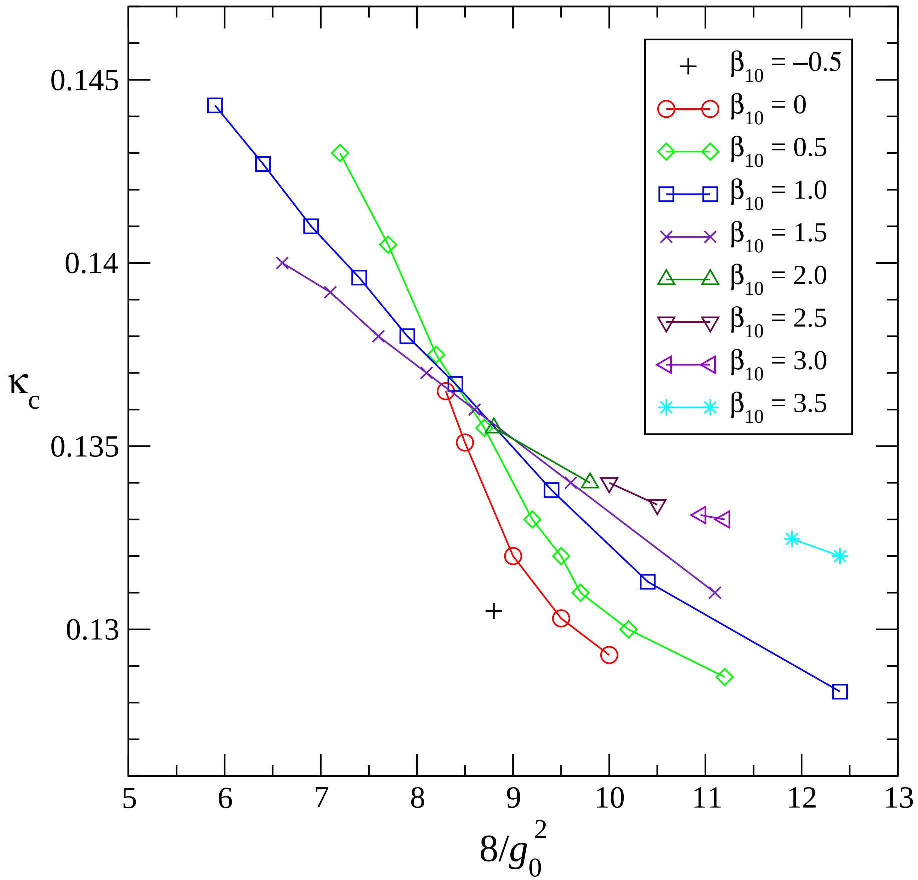




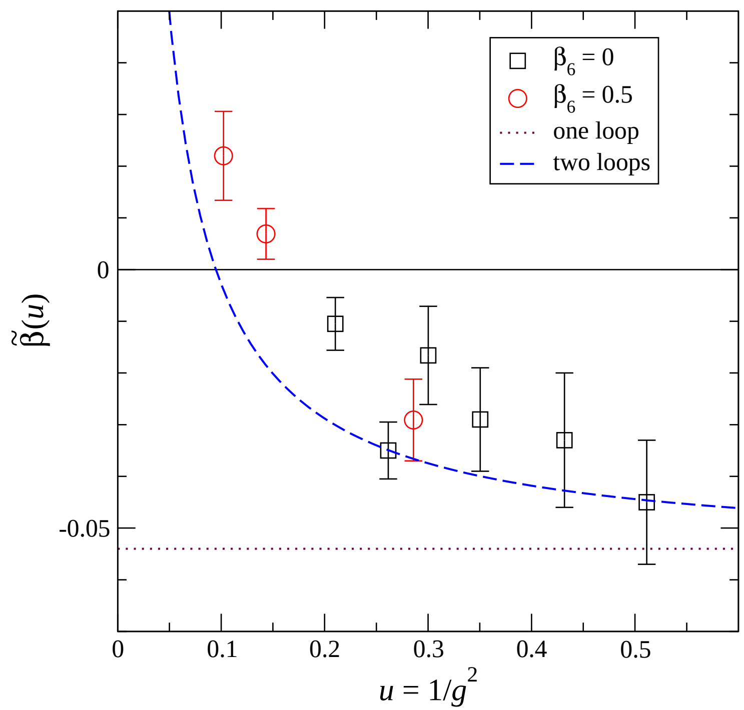
SU(4), Right to left: $\tilde{\beta} = -0.5,\ 0.0,\ 0.5,\ 1.0,\ 1.5$

Soft gauge action: κ_c

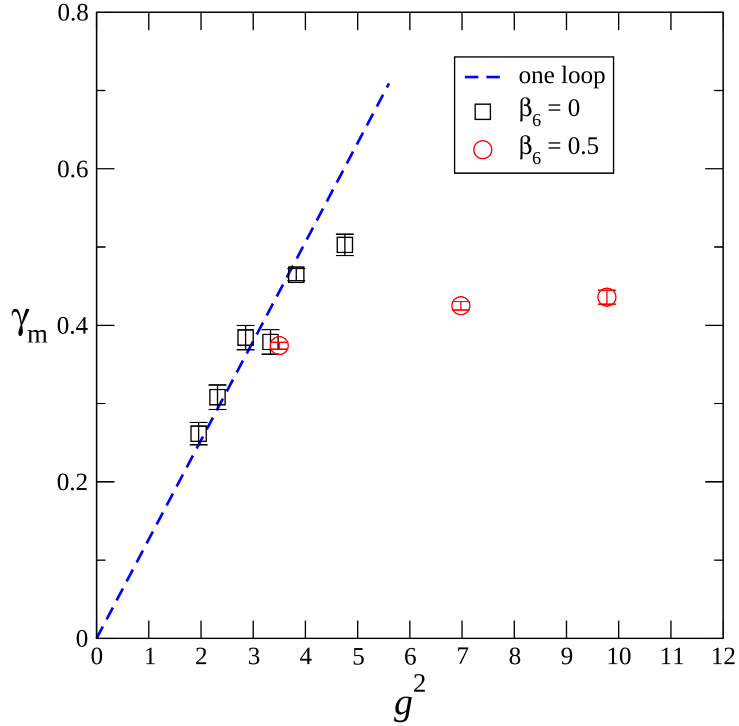
- Perturbation theory:
  κ_c increases with $\tilde{\beta}$ at fixed $1/g_0^2$
- Beyond $8/g_0^2 \approx 9$ trend reverses



Effects of the bulk transition, SU(3)



SF coupling: agreement



Mass anomalous dimension: discrepancy – due to proximity of bulk transition for $\tilde{\beta} = 0$